NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

BULLETIN

STUDY OF THE TWO-DIMENSIONAL FLOW THROUGH A CONVERGING-DIVERGING NOZZLE

By Arthur Kantrowitz, Robert E. Street, and John R. Erwin

INTRODUCTION

A study of a flow through a straight converging-diverging nozzle of simple design has been made preliminary to studies of other supersonic flows. The diverging part of the nozzle was designed by the Prandtl-Busemann method (reference 1) to give a uniform pressure at its exit of 0.298 times the initial total head, that is, to give a Mach number of 1.436. Schlieren photographs of the flow and pressure-distribution measurements along the diverging part of the nozzle were made. A comparison of the theory with these measurements is presented.

TEST APPARATUS

A source of compressed air (humidity about 0.2 percent by weight) was throttled into a settling chamber $7\frac{3}{4}$ inches in diameter, and the flow was smoothed out by passing through three 60-mesh screens. The passage was then necked down smoothly to a two-dimensional channel 1 inch wide between two pieces of thick plate glass in which the nozzle blocks were placed. The pressure ratio across the nozzle was therefore the ratio of chamber pressure to atmospheric pressure.

The schlieren photographic setup consisted of two large front-surface mirrors with the knife edge parallel to the axis of the nozzle. A high voltage spark was the source of illumination.

One of the nozzle blocks was equipped with pressure orifices in the diverging section.

DESIGN OF NOZZLE

The supersonic (diverging) part of the nozzle was designed to give a uniform pressure at the exit in a short distance in order to keep boundary-layer effects small. The design started with a minimum section of 1 inch that was held for 0.188 inch; the walls were curved until they diverged 10° (total angle) at a distance 0.328 inch from the minimum section. The angular shock (Mach) waves originating in this curved portion were then neutralized at the opposite wall by suitable local wall curvature. Typical Mach waves drawn by the method of reference 1 and the nozzle ordinates are shown in figure 1.

RESULTS

Schlieren photographs.- Schlieren photographs of typical flows at various pressure ratios are given as figure 2. The direction of the knife edge produced light regions where the density gradient was positive upward and vice versa.

At pressure ratios less than about 1.3 the flow was subsonic throughout. At pressure ratios larger than this value a supersonic flow region formed, followed by a shock wave normal to the flow. The shock wave formed first just beyond the minimum section and its position was unsteady. At higher pressure ratios the wave moved downstream and its position became steady. The shock reached the exit at a pressure ratio of about 2. The position of the normal shock measured from the photographs is plotted against pressure ratio in figure 3.

These more severe shock waves were preceded by angular compression regions and were followed by turbulent flow near the corners of the nozzle. Examination of the photographs (for example, the photograph of flow for a pressure ratio of 1.602) will show that, although there is a sharp density rise associated with the normal shock at the center of the nozzle, near the wall the rise is much smaller. Pressure distributions to be discussed subsequently support this observation.

At pressure ratios larger than 2 the normal shock moved out the end of the nozzle. Angular shock waves starting at the edges joined with a nearly normal shock in the center of the jet. The normal section of the shock shortened as the pressure ratio increased and finally disappeared at a pressure ratio of 2.6. In the pressure-ratio range between 2.6 and about 2.9, crossed shock waves formed at the end of the nozzle. At pressure ratios larger than 2.9 expansion waves appeared at the nozzle exit.

These phenomena occurring after the normal shock had left the nozzle can be readily compared with theoretical considerations. For a region where the chamber pressure was below the design value, it would be expected that the pressure at the nozzle exit would be below atmospheric and that the jet would contract on leaving the nozzle. This flow should be similar to the supersonic flow in a corner. In these experiments the initial conditions, that is, the flow before the shock, could be assumed to be known from the design of the nozzle. The pressure after the shock could be assumed atmospheric; thus the flow is determined. Similarly, if the chamber pressure is larger than the design pressure, the pressure just inside the nozzle will be above atmospheric. The jet will expand on leaving the nozzle and the flow will be similar to the flow around a corner. Both flows have been calculated in reference 2 and resulting formulas for the various wave angles and jet deflections are given in the appendix. A comparison of these theoretical results with angles measured from the schlieren photographs is given in figure 4. If the initial conditions for these calculations are taken from the measured pressure distribution as given in figure 5, the results shown as dot-dash lines in figure 4 are obtained. The agreement between theory and experiment is considered satisfactory.

Pressure distribution.- The pressure distribution in the diverging section of the nozzle is shown in figure 5 for a series of chamber pressures. At the lowest pressure the flow was entirely subsonic. At higher pressures the pressure rose smoothly through the region of the shock wave in spite of the fact that the schlieren photographs showed a sharp steady normal shock. This may be due to the action of local turbulent flows in the vicinity of the intersection of the shock wave and the walls. There is also some reason for believing that the boundary layers in the corners exerted an important influence on these pressure distributions.

At still higher pressures, after the normal shock had disappeared, the nozzle began to approach the theoretical pressure distribution. The remaining discrepancy at the highest chamber pressures is to be attributed to the nozzle boundary layer.

CONCLUSION

With allowances for boundary-layer effects, the experimental results obtained from the pressure-distribution measurements of supersonic flow through a straight converging-diverging nozzle after the normal shock had been blown out of the nozzle were in close agreement with conventional theory. Readings from pressure orifices in the nozzle walls showed a smooth rise in pressure in the region of the normal shock, even though the position of the shock as determined from the schlieren photographs was quite steady.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va.

APPENDIX

FLOW IN AND AROUND CORNERS

Meyer's theory of supersonic flow inside a sharp corner is described in reference 2 as flow through an oblique shock. Although the derivation of these equations is not presented, the results are given herein. With the ratio of the heat capacities $\gamma = 7/5$ for air, the equation for angle A in figure 4 is

$$\cos^2 A = \frac{1}{7M^2}\left(1 + 6\,\frac{p_a}{p_1}\right)$$

where

$$\frac{p_a}{p_1} = \frac{p_c}{p_1} \times \frac{p_a}{p_c} = 3.356\,\frac{p_a}{p_c}$$

and

M Mach number

p_a atmospheric pressure

p_1 pressure just before shock

p_c chamber pressure

From the nozzle design $\frac{p_1}{p_c} = 0.298$, M = 1.436. The equation for angle C is

$$\cot C = \frac{\frac{p_a}{p_1} - 1}{3.891 - \frac{p_a}{p_1}} \tan A$$

For flow around a corner, the pressure ratio at any radius which makes an angle θ to a fixed radius is given in reference 2 as

$$\left(\frac{p}{p_c}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{\gamma + 1} (1 + \cos 2\lambda\theta)$$

with $\lambda = \sqrt{\frac{\gamma - 1}{\gamma + 1}}$. The complement of the Mach angle m is denoted by ψ and is obtained from

$$\tan \psi = \frac{1}{\lambda} \tan \lambda\theta$$

The angle ν turned through by the stream relative to the flow at $\theta = 0$ is then $\nu = \theta - \psi$. A graph was made of θ, ψ, and ν against p/p_c from these relations for $\gamma = 7/5$ (fig. 6).

The initial conditions are fixed for the case of exit pressures above atmospheric at $\frac{p_1}{p_c} = 0.298$. The corresponding values of θ, ψ, and ν are read from the graph as

55.9°, 45.9°, and 10.0°, respectively. As a result, the theoretical values of ψ, B, and C shown in figure 4 are ψ = constant = 45.9°.

$$B = \theta_a - \theta + \psi = \theta_a - 10^\circ$$

and

$$C = 90^\circ + (\nu_a - \nu) = 80^\circ + \nu_a$$

The values of θ_a and ν_a, where the subscript a is used to indicate that θ and ν are functions of p_a/p_c, are read from the same graph.

REFERENCES

1. Busemann, A.: Gasdynamik. Handbuch der Experimentalphysik, Bd. IV, 1. Teil, Akad. Verlagsgesellschaft m. b. H. (Leipzig), 1931, pp. 341-460.

2. Taylor, G. I., and Maccoll, J. W.: The Mechanics of Compressible Fluids. The Two-Dimensional Flow Around a Corner. Vol. III, div. H, sec. 5, ch. IV of Aerodynamic Theory; W. F. Durand, ed.; Julius Springer (Berlin), 1935, pp. 243-246.

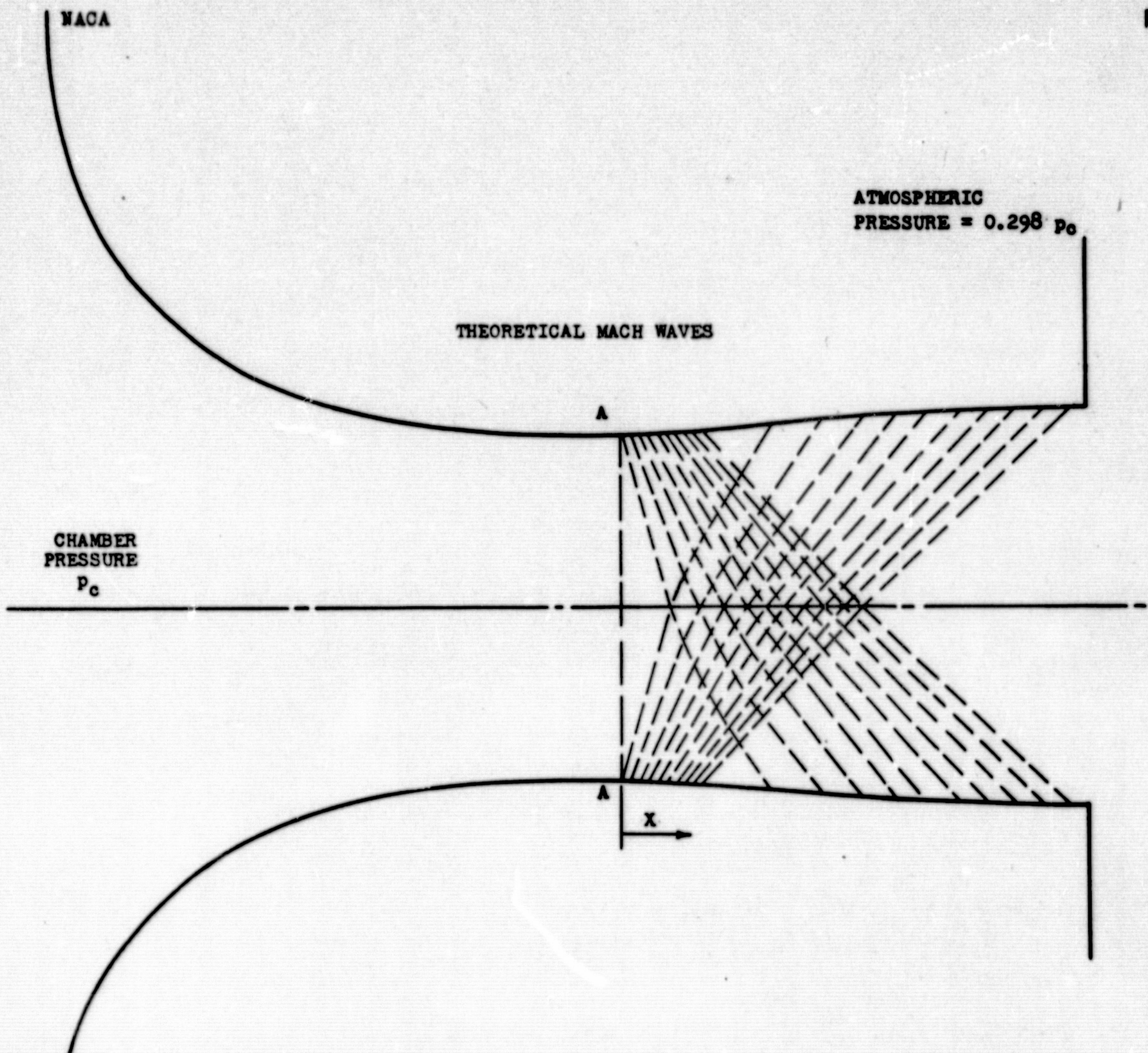

Figure 1.- NOZZLE DESIGN.

CROSS SECTION S OF NOZZLE AS A FUNCTION OF DISTANCE
X MEASURED FROM REFERENCE LINE A-A AT
SUPERSONIC END OF MINIMUM SECTION

X (in.)	S (in.)	X (in.)	S (in.)
-1.658	3.238	0.126	1.0066
-1.618	2.680	0.151	1.0092
-1.538	2.266	0.176	1.0122
-1.418	1.896	0.202	1.0158
-1.258	1.576	0.228	1.0198
-1.058	1.316	0.429	1.0550
-0.858	1.164	0.575	1.0780
-0.658	1.076	0.691	1.0942
-0.458	1.024	0.789	1.1062
-0.258	1.002	0.876	1.1154
-0.188	1.0000	0.970	1.1236
0	1.0000	1.050	1.1292
0.024	1.0004	1.128	1.1332
0.049	1.0012	1.211	1.1362
0.075	1.0026	1.292	1.1374
0.101	1.0044	1.342	1.1374

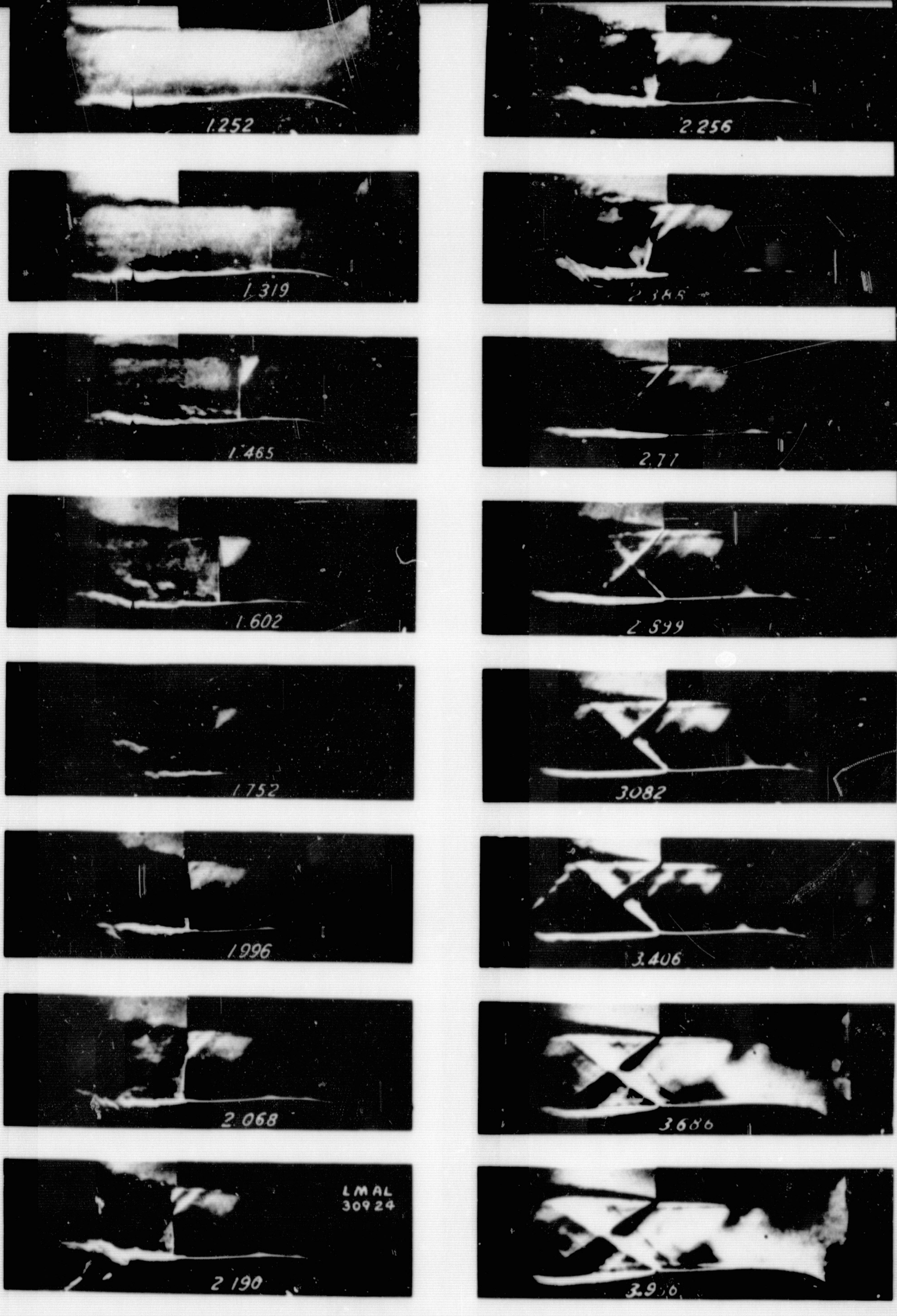

FIGURE 2. - SCHLIEREN PHOTOGRAPHS OF FLOW THROUGH THE NOZZLE. NUMBERS ARE CHAMBER PRESSURE IN ATMOSPHERES.

NACA

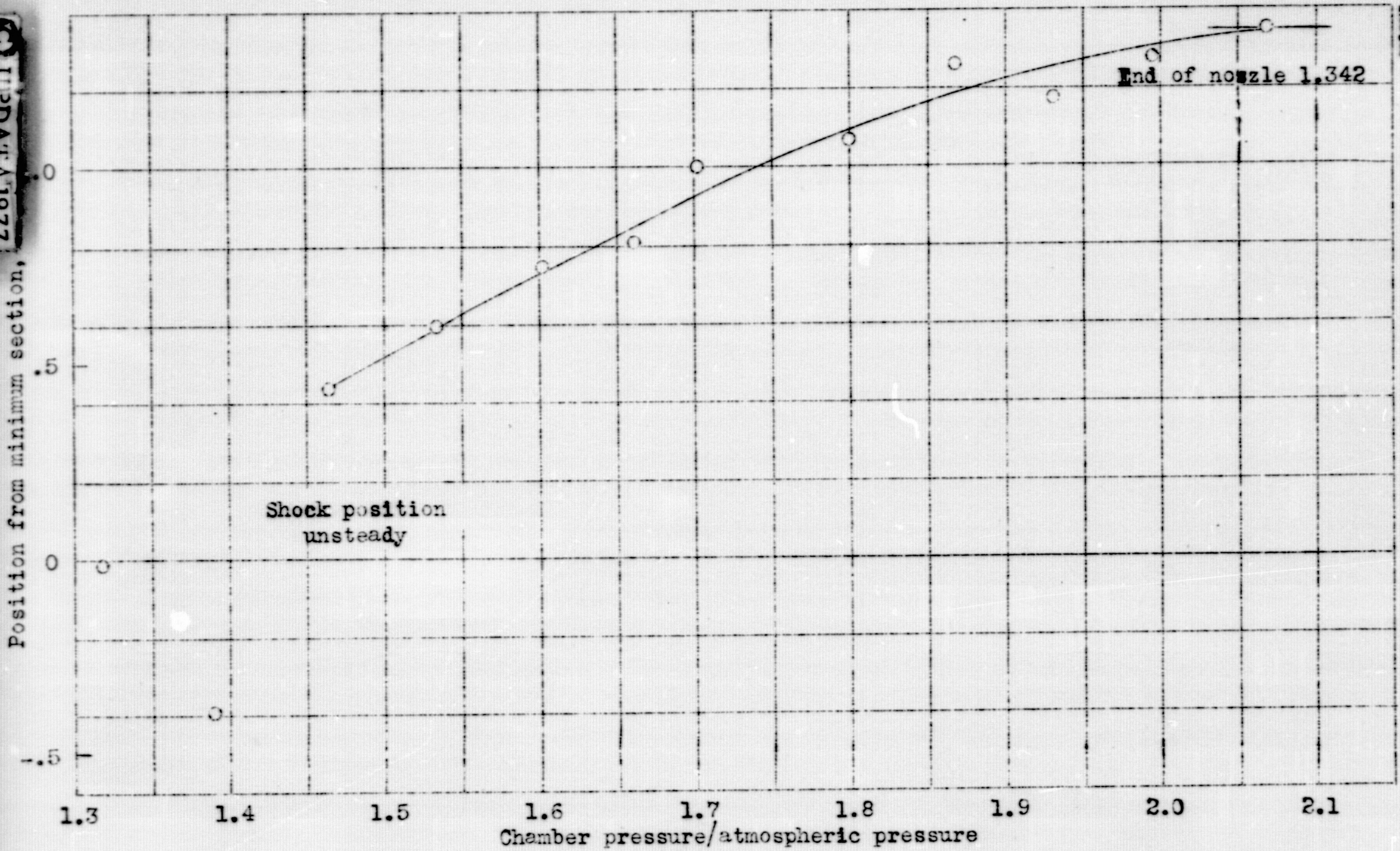

Figure 3.- Position of normal shock in nozzle measured from Schlieren photographs.

Fig. 3

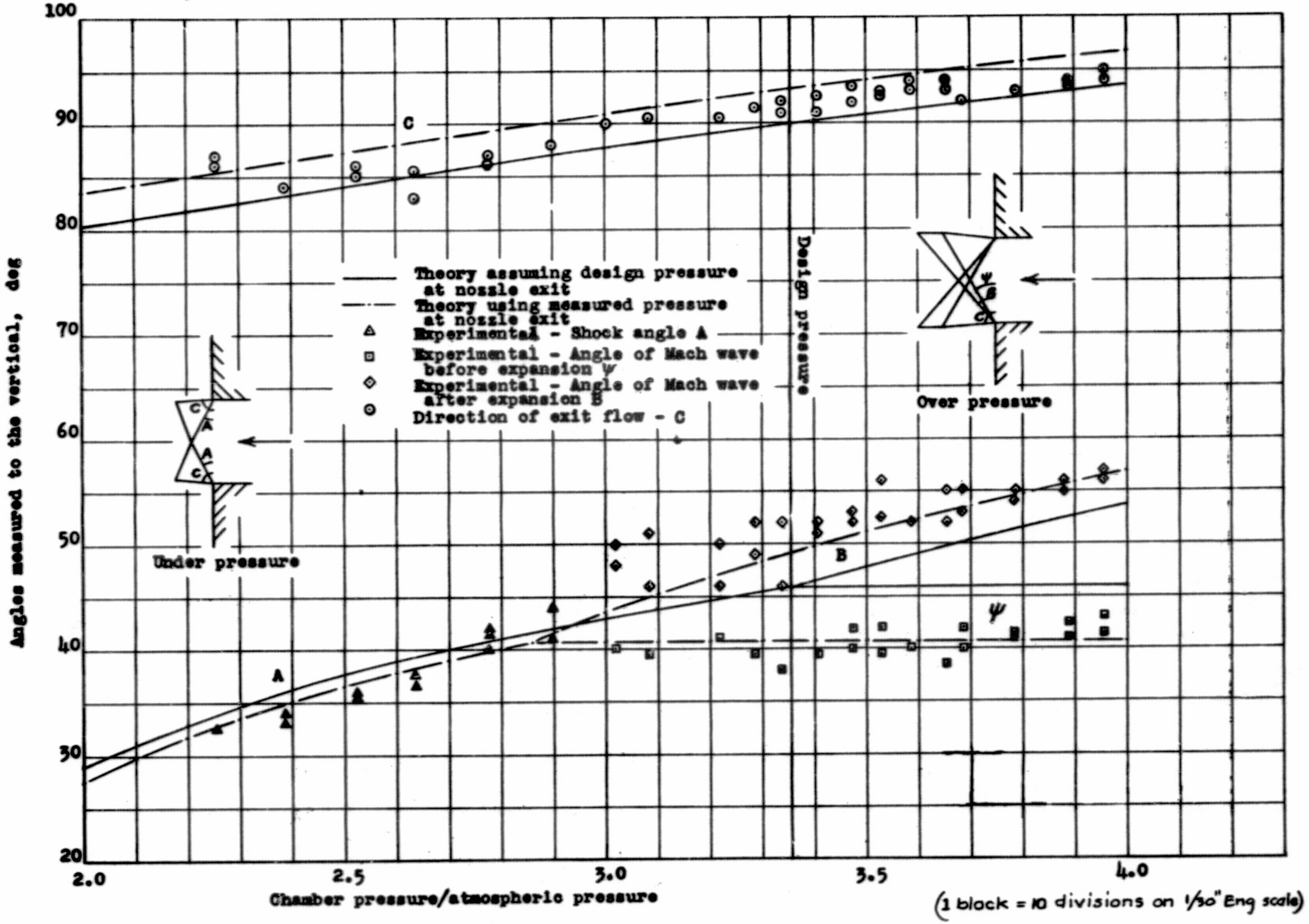

Figure 4.- Comparison of theory with various measured angles at nozzle exit.

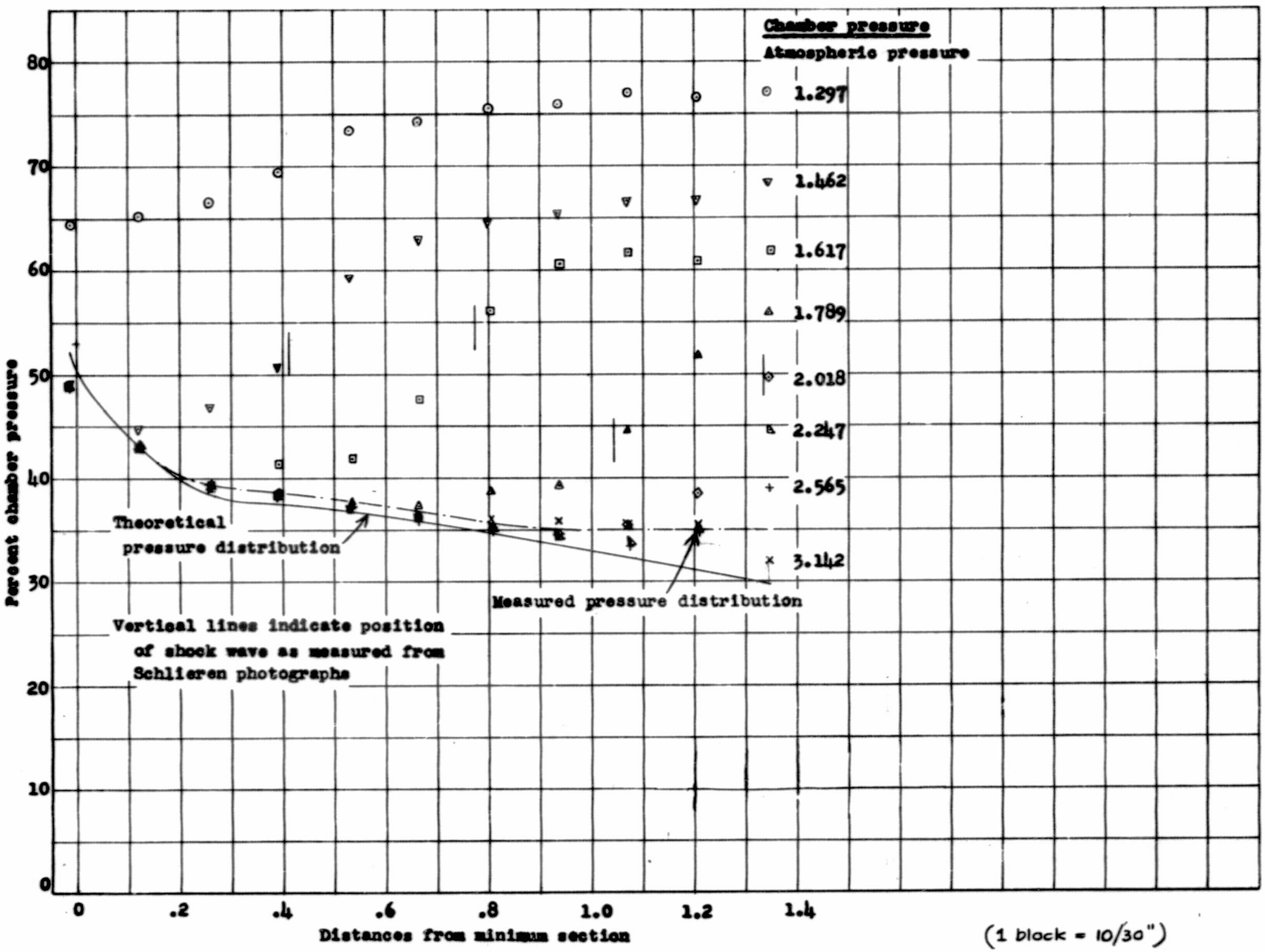

Figure 5.- Pressure distribution in diverging section of nozzle.

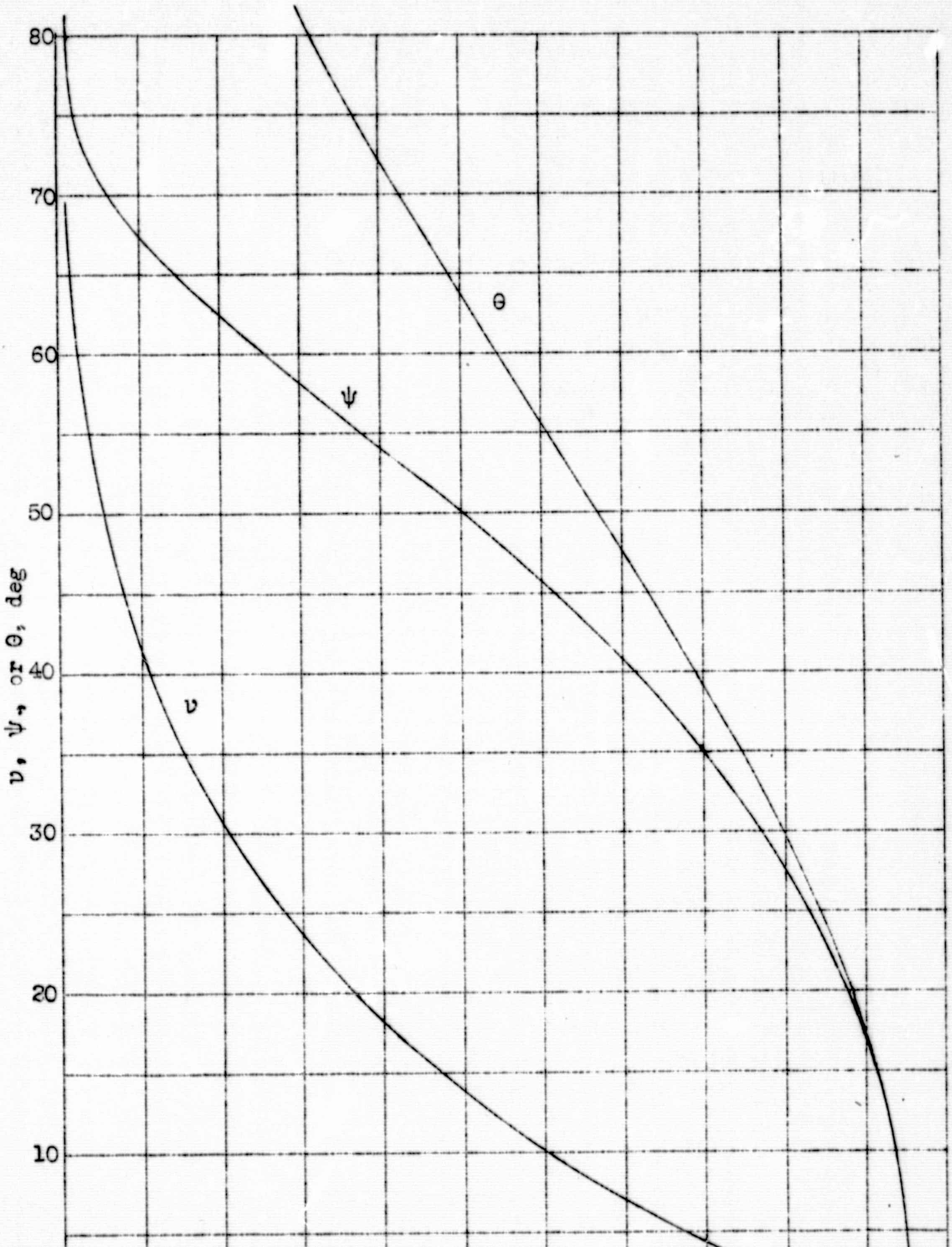

Figure 6.- Angles for expansion around a corner.